# Shift Characteristics Analysis and Smooth Shift for an Automatic Power Transmission

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Smooth shift is one of the key issues in automatic power transmission control systems. However, the torque sensors are too expensive to be used in shift controllers on production vehicles. In order to provide a basic strategy for smooth shifting by using RPM sensors only and in order to accomplish the shift within a designated time, this paper studies detailed characteristics of the smooth shift for clutch-to-clutch shift mechanism. A desired trajectory of slip speed is proposed for smooth acceleration shift defined in this paper. Also the clutch torque needed to achieve this trajectory is derived, and it may be used as a open loop shift control law.

Key Words : Clutch-to-Clutch Shift, Smooth Shift, Engaged Clutch Torque, Indirect Acceleration Shift, Heat Dissipation, Dynamic Shift Condition

# 1. Introduction

In spite of the lower efficiency and higher cost, automatic power transmission system has gained widespread use in most passenger vehicles due to its easy drive-ability, as well as torque amplifying and the torsional damping characteristics of torque converter. In shift control, two important issues are shift point decision and shift quality control (Jeong, 1993). The former considers the issue of under what operating condition the shift should be conducted for greater fuel economy. The latter is concerned with how to accomplish gear ratio changes smoothly for passenger comfort. This paper concentrates on the shift quality control.

A conventional automatic transmission uses planetary gear sets to provide different speed ranges and over-running sprags to give smooth transitions between the speed ranges. If a smooth clutch-to-clutch shift were possible, this expensive mechanical component can be eliminated and the overall kinematic arrangement can be significantly simplified. This step offers economic advantages but presents a challenging control problem. Two primary objectives in shift control are smooth transients and fast shift completion. The shift smoothness is related to the vehicle acceleration, which is proportional to the torque of the drive axle shaft. The fast shift completion is related to the clutch energy dissipation and components' longevity. Thus, this paper presents methods for effecting a smooth shift within a short time period.

Meanwhile, the sensors currently used for measuring the shaft torque are too expensive to be used on production vehicles for control purpose, but magnetic pick-up speed sensors can be widely applied for speed checking, shift decision, engine control and anti-skid braking, etc. Thus, indirect means for regulating shift torque using speed information is required.

Recalling a long history and wide popularity of automatic transmission, there are plenty of experimental results on the shift characteristics and shift

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control design. However, theoretical analysis is rather rare to find in the literature. Cho and Hedrick (1989a) developed the power-train model for control. The modeling effort was directed to achieving a reasonable trade-off between the simplicity and the comprehensiveness of the model comprising three major components, i. e. engine, transmission and drive-train. Torque estimation of the vehicle axle shafts was studied by use of inexpensive RPM sensors to facilitate a nonlinear control algorithm (Masmoudi and Hedrick, 1992) that applied the sliding mode theory developed for state observation (Slotine, et al., 1987). A sliding mode control was designed for clutch-to-clutch shift and an alternative formulation to deal with uncertainties in actuator or control dynamics was presented (Cho and Hedrick, 1989b). The speed gaps between the desired and the actual value of reaction carrier and torque converter turbine were set up as two sliding surfaces. But there was no mention about the desired speed trajectories, which may be more important in some sense. Jeong sought an indirect shift control using speed sensors and suggested the desired slip speed trajectory (Jeong and Lee, 1994a; 1994b). This paper refines the previous idea and proposes clutch slip speed for indirect smooth acceleration shift.

The remainder of this paper is organized as follows: Section 2 describes the system model of clutch-to-clutch shift. Under some simplifying assumptions, Sec. 3 analyzes the shift characteristics and three types of smooth shift are discussed in Sec. 4. In the following section, methods for achieving smooth acceleration shift indirectly without measuring torque information are presented. Conclusions will be followed in the last section.

## 2. Clutch-to-Clutch Shift Model

A typical clutch-to-clutch shift model of an automatic transmission may be depicted as in Fig. 1. It changes speed ratio by disengaging the upper (old) clutch and by engaging the lower (new) clutch in the figure. The system consists of a engine, torque converter, gear sets, final drive gear, drive axle shaft and load torque. Engine modeling was studied in great depth. Refer to Cho and Hedrick (1989a) and references contained therein. A torque converter consists of three elements such as the pump (driving member), turbine (driven member) and stator (reaction member) and it can be modeled with various complexity. The advantages of the torgue converter in vehicle applications are the damping characteristics isolating vibrational torque from engine firing pulses, and the torque amplifying characteristics at the torque mode desirable for vehicle acceleration, while its fluid coupling mode prevents an engine from stalling under excessive load. Generally, the lock-up clutch attached inside of a torque converter housing is disengaged during shift. Thus, the torque converter dynamics should be involved in the shift analysis. Two-states static model of Kotwicki (1982) is desirable due to its simplicity and four -states dynamic model of Hrovat and Tobler (1985) may be suitable for more detailed studies of power-train dynamics.

The drive train includes several planetary gear sets, wheel inertias, stiffnesses of axle shafts, longitudinal inertias of vehicle, etc. In this paper, the axle shaft is approximated as a lumped torsional



Fig. 1 Shift model of automatic transmission

spring as in Eq. (5) (Cho and Hedrick, 1989a) and the details on the engine, torque converter and load torque are not included. The above power transmission can be given in mathematical form as follows

$$I_{EP}\dot{\omega}_P = T_{ENG} - T_P \tag{1}$$

$$I_{TG}\dot{\omega}_{T} = T_{T} - \left(\frac{1}{r_{o}}T_{Co} + \frac{1}{r_{n}}T_{Cn}\right)$$
(2)

$$I_M \dot{\omega}_M = (T_{Co} + T_{Cn}) - \frac{1}{\gamma_F} T_s \tag{3}$$

$$I_V \dot{\omega}_V = T_s - T_L \tag{4}$$

$$\dot{T}_{s} = K_{s} \left( \frac{1}{r_{F}} \omega_{M} - \omega_{V} \right), \tag{5}$$

where subscript o means the old (off-going) clutch, n the new (up-coming) clutch, and F the final drive, respectively. And others stand for as denoted in the figure. Speed relations before and after the new clutch engagement are

before: 
$$\omega_T = r_o \omega_{Co} = r_o \omega_M$$
 (6)  
after:  $\omega_T = r_n \omega_{Cn} = r_n \omega_M$ 

From Eqs. (2) and (3), the corresponding governing equations yield

before : 
$$(r_o^2 I_{TG} + I_M) \dot{\omega}_M = r_o T_T - T_s/r_F$$
  
after :  $(r_n^2 I_{TG} + I_M) \dot{\omega}_M = r_n T_T - T_s/r_F$ 
(7)

Throughout the paper, the old clutch torque  $T_{co}$  will be assumed to be cut off instantaneously as the shift starts. Let us denote the desired shift completion time by  $\tau$  and occasionally denote the new clutch torque  $T_{cn}$  as u(t) to emphasize that it is an external input variable.

### 3. Shift Characteristics Analysis

In this and the following sections, for analytical simplicity we assume that the turbine torque and load are constant during the shift and that the drive axle shaft is rigid. Then, the system equations are

$$I_{TG}\dot{\omega}_{T} = T_{T} - \left(\frac{1}{r_{o}}T_{co} + \frac{1}{r_{n}}T_{cn}\right)$$
(8a)

$$I_{MV}\dot{\omega}_{M} = (T_{Co} + T_{Cn}) - \frac{1}{\gamma_{F}} T_{L},$$
  

$$I_{MV} = I_{M} + I_{V} / r_{F}^{2}$$
(8b)

$$\omega_M = \gamma_F \omega_V \tag{8c}$$

In the following, we investigate several impor-

tant shift characteristics under the assumptions mentioned.

#### 3.1 Slip speed $\omega_s$

The slip speed of new clutch  $\omega_s = \omega_{Cn} - \omega_M$ may be obtained as

$$\dot{\omega}_{s} = \left(\frac{1}{r_{n}I_{TG}}T_{T} + \frac{1}{I_{MV}}\frac{T_{L}}{r_{F}}\right) - \left\{\left(\frac{1}{r_{n}^{2}I_{TG}} + \frac{1}{I_{MV}}\right)T_{Cn} + \left(\frac{1}{r_{n}r_{o}I_{TG}} + \frac{1}{I_{MV}}\right)T_{Co}\right\}\right\} (9)$$
$$= \alpha_{n} - \beta_{n}T_{Cn} - \beta_{no}T_{Co}$$

where the equivalent accelerations  $a_i$ , and  $\beta_i$  are defined as

$$\alpha_{i} = \frac{1}{r_{i}I_{TG}}T_{T} + \frac{1}{I_{MV}}\frac{T_{L}}{r_{F}}, \ i = o, \ n \tag{10}$$
$$\beta_{i} = \frac{1}{r_{i}^{2}I_{TG}} + \frac{1}{I_{MV}}, \ \beta_{no} = \frac{1}{r_{n}r_{o}I_{TG}} + \frac{1}{I_{MV}}$$

Introducing new notations, the inertia ratio  $\gamma_{Ii}$ and the torque ratio $\gamma_{Ti}$  can be expressed as

$$\gamma_{Ii} = \frac{r_i^2 I_{TG}}{I_{MV}} < 1, \ \gamma_{Ti} = \frac{T_L/r_F}{r_i T_T} < 1 \quad i = 0, \ n$$
(11)

Then, the equivalent accelerations  $\alpha_i$  and  $\beta_i$ may be given by

$$\alpha_i = \frac{1 + \gamma_{Ii} \gamma_{Ti}}{\gamma_{Ii}} \frac{r_i T_T}{I_v}, \ \beta_i = \frac{1 + \gamma_{Ii}}{\gamma_{Ii}} \frac{1}{I_v}$$

Equations (9)  $\sim$  (11) shows that the clutch slip speed depends on the equivalent acceleration term and the torques applied to the clutches. Initial slip speed of new clutch  $\omega_{so}$  at the start of shift can be given by

$$\omega_{so} = \frac{r_o - r_n}{r_o r_n} \omega_{r_o} = \frac{r_o - r_n}{r_n} \omega_{Mo}$$
(12)

Integrating the slip speed Eq. (9), with the assumption that  $T_{co}=0$  after the shift starts,

$$\omega_s(t) = \omega_{so} + \alpha_n t - \beta_n \int u(t) dt \qquad (13)$$

Considering that the slip speed becomes zero at time  $t=\tau$ , one may obtain Eq. (14a). But in general, the equivalent acceleration  $\alpha_n$  is not constant. Thus, in this case it should be modified as in Eq. (14b). For the slip speed to decrease monotonically, note that an additional condition given by Eq. (14c), should hold.

$$\omega_s(\tau) = \omega_{so} + \alpha_n \tau - \beta_n \int_o^\tau u(t) dt = 0 \qquad (14a)$$

$$\omega_s(\tau) = \omega_{so} + \int_0^\tau \alpha_n dt - \beta_n \int_o^\tau u(t) dt = 0$$
(14b)

$$T_{Cn} > \alpha_n / \beta_n \tag{14c}$$

After the shift is completed, the clutch torque always satisfies either the conditions (14a) or (14b). Thus, Eqs. (14a) and (14b) will be called the *static shift condition and dynamic shift condition*, respectively. And Eq. (14c) will be called the *monotonic shift condition*.

#### 3.2 Vehicle acceleration $\alpha_{\nu}$

Before or after the shift, the vehicle acceleration may be derived from Eqs. (8a)-(8b) as follows

$$r_{F}\alpha_{Vi} = \alpha_{Mi} = \frac{r_{i}T_{T} - T_{L}/r_{F}}{r_{i}^{2}I_{TG} + I_{MV}}$$
$$= \frac{1 - r_{Ti}}{1 + r_{Ii}} \frac{r_{i}T_{T}}{I_{MV}}, \ i = 0, \ n \tag{15}$$

During shift, note that the vehicle acceleration depends on the load and clutch torque. It has no relationship with the turbine torque. From Eq. (15), one can derive an inequality  $sign(r_o - r_n)$  $(a_{vo} - a_{vn}) > 0$  with  $I_{MV} \gg r_n r_o I_{TG}$ . Usually the vehicle inertia is much larger than that of other components. Thus, the inequality shows that the vehicle acceleration always decreases (increases) during up-shift (down-shift).

#### 3.3 Engaged clutch torque $T_E$

While the clutch torque during the shifting period is decided by the frictional force, it was pointed out that the torque of engaged clutch after the shift is determined by completely different principle (Jeong and Lee, 1998). When the clutch is engaged, the slip speed and its derivative are both zero. Thus, the following *engaged clutch torque* can be obtained from Eq. (9)

$$T_{Ei} = \frac{r_i^2 I_{TG} I_{MV}}{r_i^2 I_{TG} + I_{MV}} (\frac{1}{r_i I_{TG}} T_T + \frac{1}{I_{MV}} \frac{T_L}{r_F}),$$
  
 $i = 0, n$  (16)

Using definitions (10) and (11), the engaged clutch torque may yield

$$T_{Ei} = \frac{\alpha_i}{\beta_i} = \frac{1 + \gamma_{Ii}\gamma_{Ti}}{1 + \gamma_{Ii}} \gamma_i T_T, \ i = 0, \ n$$
(17)

Note that the engaged clutch torque depends

both on the turbine torque and load torque. Also note that  $sign(r_o - r_n)(T_{Eo} - T_{En}) > 0$  can be obtained from Eq. (16) with  $I_{MV} \gg r_n r_o I_{TG}$ . Thus, the engaged clutch torque always decreases (increases) during up-shift (down-shift). Also the following meaningful inequalities can be derived

$$sign(r_{o}-r_{n})(\alpha_{o}-\alpha_{n}) < 0,$$
  

$$sign(r_{o}-r_{n})\left(\frac{1+\gamma_{Io}}{\gamma_{Io}}-\frac{1+\gamma_{In}}{\gamma_{In}}\right) < 0$$
  

$$sign(r_{o}-r_{n})(\beta_{o}-\beta_{n}) < 0,$$
  

$$sign(r_{o}-r_{n})\left(\frac{1+\gamma_{Io}\gamma_{To}}{\gamma_{Io}}-\frac{1+\gamma_{In}\gamma_{Tn}}{\gamma_{In}}\right) < 0$$

#### 3.4 Heat dissipation during shift

Heat dissipation is closely related with clutch wear and oil temperature elevation. Thus, the amount of the heat dissipated is important for components' longevity. The dissipated heat H(t)can be obtained from the difference between total input energy and output energy of the clutch as follows

$$H(t) = \int T_c (\omega_i - \omega_o) dt = \int T_c \omega_s dt \quad (18)$$

Substituting the slip speed Eq. (13), one obtains the following

$$H(t) = \omega_{so} \int u(t) dt + \alpha_n t \int u(t) dt$$
$$-\frac{\beta_n}{2} \left[ \int u(t) dt \right]^2 - \alpha_n \iint u(t) dt dt$$

Applying the static shift condition to this equation, total dissipated heat during the shift becomes

$$H(\tau) = (\omega_{so} + \alpha_n \tau) \int_0^\tau u(t) dt - \frac{\beta_n}{2}$$
$$\left[ \int_0^\tau u(t) dt \right]^2 - \alpha_n \int_0^\tau \int_0^t u(t) dt dt$$
$$= \frac{1}{\beta_n} [(\omega_{so} + \alpha_n \tau) (\omega_{so} + \alpha_n \tau - \frac{1}{2})]$$
$$- \alpha_n \int_0^\tau \int_0^t u(t) dt dt \qquad (19)$$

Note that the first term of Eq. (19) mainly depends on the shift completion time  $\tau$  and the second term is a function of the applied clutch torque. Therefore, the shift should be completed as soon as possible. Now, let us consider what is the optimal clutch torque profile in view of the



Fig. 2 The second heat dissipation term U(t)dissipated energy. Defining a non-decreasing function  $U(t) = \int_{0}^{t} u(t) dt$  and recalling that U  $(\tau)$  is constant for the static shift condition, (14a), one can postulate four typical cases of the clutch torque profile as shown in Fig. 2. In the figure, Case (1) maximizes  $U(\tau)$  i. e, minimizes the total heat dissipation  $H(\tau)$ . However this case is unrealistic since it implies a sudden impulse shift. Case (2) represents a realistic suboptimal case for minimizing the heat dissipation. Note that u(t) for this case decreases with time starting from the high initial value. Case (3) means a constant clutch torque and (4) is the worst case in view of the heat dissipation. Unfortunately, the commonly adopted clutch pressure profile during production is the worst Case (4) in which the pressure increases with time.

# 4. Smooth Shift

Following the discussions of the last section, the system parameters before and after shift are different. The degree of freedom during shift also increases by one compared with the degree of freedom after the shift. Moreover, several variables such as input turbine torque and load, etc. can vary with time depending on the operating conditions.

For the passenger comfort, a smooth transient during shift is essential. To this end, one has to enforce the system to progress from the initial conditions (before shift) to the final states (after shift) without abrupt variations. During this process, a dynamic shift condition, Eq. (14b), should be met. For components' longevity, shift must be accomplished within a suitably short period. Recalling these facts, we seek how to make smooth transient within a designated shift completion time  $\tau$ .

#### 4.1 Shift types and smooth curve

One may postulate three kinds of state transition as follows. Shift is conducted by disengaging one clutch and by engaging another clutch with different ratios. *Smooth slip (speed) shift occurs* first, which tries to make the slip speed of new clutch change from its initial speed to zero in a smooth manner.

During shift, the clutch torque is decided entirely by the applied pressure. However, the engaged clutch torque is independent of the actuating pressure. It generally decreases after upshift. So, changing the clutch torque from old engaged torque,  $T_{Eo}$ , to new engaged torque,  $T_{En}$ , smoothly while satisfying the shift condition may accomplish the shift process. This will be called the *smooth* (engaged clutch) torque shift.

Substituting Eqs. (8b)-(8c) to Eq. (14b), a dynamic shift condition in terms of the vehicle acceleration gives

$$\int_{0}^{\tau} \alpha_{V}(t) dt = \frac{1}{\beta_{n} \gamma_{F} I_{MV}} \left\{ \omega_{so} + \int_{0}^{\tau} (\alpha_{n} - \beta_{n} \frac{T_{L}}{\gamma_{F}}) dt \right\}$$
(20)

The vehicle acceleration before and after the shift is given by Eq. (15). It generally decreases after upshift. Thus, moving the vehicle from its initial acceleration to the final state smoothly while satisfying condition (20) will be called the *smooth* (*vehicle*) acceleration shift.

Now, let us consider what is the smooth transition from one state to another while satisfying the shift condition. In the context of variational calculus, the problem may be written

find 
$$\min_{y} \int_{0}^{\tau} \sqrt{1 + {y''}^2} dt$$
 subject to  
 $\int_{0}^{\tau} y(t) dt = A$ 

It is difficult to find the analytical solution for this problem. As an approximation of a smooth function solution, cycloidal curve (21) may be chosen. It is shown in Fig. 3(a) where  $y_o$  and  $y_n$ 



Fig. 3(a) Smooth curve y(t)



Fig. 3(b) Combined smooth curve  $y_c(t)$ 



Fig. 3(c) Delayed smooth curve  $y_d(t)$ 

denote two end points.

$$y(t) = \begin{cases} y_o - r(t - \frac{s}{\pi} \sin \frac{\pi}{s}t), & 0 < t < s \\ y_o - 2rt + rs, & s < t < \tau - s \\ y_o - r[(t + \tau - 2s) - \frac{s}{\pi} \sin \frac{\pi}{s}(t - \tau + 2s)], & \tau - s < t < \tau \end{cases}$$
(21)

The curve parameters must satisfy  $y_o - y_n = 2r$  $(\tau - s)$ . The area under the curve is  $A = (y_o + y_n)$  $\tau/2$ . If the required area is bigger or much smaller than A, two curves may be combined, denoted by  $y_c(t)$  in Fig. 3(b). If the required area is moderately smaller than A, delayed smooth curve,  $y_d$ (t) shown in Fig. 3(c) can be adopted as a desired state trajectory.



Fig. 4(c) Smooth acceleration shift

#### 4.2 Simulation results

Following the above discussions, a simple cycloid curve,  $\omega_s(t) = y(t)$ , was selected as a desired trajectory for smooth slip shift. From Eq. (9), the clutch torque for this shift can be given by  $u(t) = (\alpha_n - y'(t))/\beta_n$ .

Desired clutch torque for smooth torque shift is chosen so that  $u(t) = y_c(t)$  or  $u(t) = y_d(t)$ . The corresponding slip speed and vehicle acceleration during the shift may be obtained from Eq. (9) and Eq. (8b), respectively. Desired trajectory for the smooth acceleration shift will be either  $a_V(t)$  $= y_d(t)$  or  $a_V(t) = y_c(t)$  depending on the desired shift completion time and shift condition. The clutch torque for effecting this shift can be determined from Eq. (8b).

Simulation results of each shift are shown in Fig. 4 for various curve parameters. A shift completion time was set at 3 sec. for visualization purpose. However, short shift time does not siginficanfly affect the response. Desired curve for smooth torque and acceleration shift may be selected according to the area requirement based on the required shift completion time and the shift condition. All of the plots in the figure show smooth transients except for the initial jerk at the beginning of the smooth slip shift. It is due to the fact that the initial clutch torque,  $u(0) = \alpha_n / \beta_n =$  $T_{En}$ , of this shift is less than the initial engaged clutch torque  $T_{Eo}$ . Note that the torque shift and acceleration shift show the same effect even though their concepts are different. The reason is that the clutch torque directly drives vehicle inertia, which in turn induces vehicle acceleration.

# 5. Indirect Smooth Acceleration Shift

Upon inspection of the slip speed and vehicle acceleration shown in Fig. 4, a possibility that smooth acceleration shift may be achieved by adjusting slip speed trajectory is worth investingating. The following equation may be derived from Egs. (8b) and (9).

$$I_{MV}\beta_n r_F \alpha_V = \frac{1}{r_n I_{TG}} T_T - \frac{1}{r_n^2 I_{TG}} \frac{T_L}{r_F} - \dot{\omega}_s \quad (22)$$

The slip speeds and derivatives for various shift completion times are shown in Fig. 5. Thus, the figure and Eq. (22) justify the above conjecture.

# 5.1 Desired slip speed for indirect smooth acceleration shift

Under the simplifying assumptions that the turbine and load torque is invariant, a smooth acceleration shift may be carried out by setting the desired slip acceleration  $\dot{\omega}_s^o(t)$  as a smooth curve with area  $A = \omega_{so}$  and with two end points  $y_n = \dot{\omega}_s(\tau) = 0$  and  $y_o = \dot{\omega}_s(0) = -\beta_n (T_{Eo} - T_{En})$ , respectively.

However, the turbine torque and load are not constant during shifting. To compensate these variations, the desired slip  $\dot{\omega}_s^o$  of the invariant torque case must be modified as



Fig. 5 Slip speed curves for indirect smooth shift

$$\dot{\omega}_{s}^{*} = \dot{\omega}_{s}^{o} + \frac{1}{r_{n}I_{TC}}(T_{T} - T_{To}) - \frac{1}{r_{n}^{2}I_{TC}}\frac{(T_{L} - T_{Lo})}{r_{F}}$$
(23)

where  $T_{To}$  and  $T_{Lo}$  are the torques at the moment the shifting starts. New slip curve  $\dot{\omega}_s^*$  makes the vehicle acceleration of time varying torque case the same as that of the constant torque case.

Moreover, the torsional spring effect of the axle shaft should be considered. To this end, a slip speed-vehicle acceleration relationship such as Eq. (22) may be derived from Eqs. (2)-(4) and the modified curve reflecting this effect can be given by

$$\dot{\omega}_{s}^{*} = \dot{\omega}_{s}^{o} + \frac{1}{r_{n}I_{TG}}(T_{T} - T_{To}) + \frac{1}{r_{F}I_{M}}(T_{s} - T_{so}) - \left(\frac{1}{r_{n}^{2}I_{TG}} + \frac{1}{I_{M}}\right) \left\{\frac{1}{r_{F}}(T_{L} - T_{Lo}) + I_{M}(\dot{\omega}_{M} - \dot{\omega}_{Mo})\right\}$$

In noisy automotive environment however, this curve is undesirable because it contains the derivative terms. For the subsequent analysis, it can be resolved simply by replacing the load torque of Eq. (23) with the shaft torque

$$\dot{\omega}_{s}^{*} = \dot{\omega}_{s}^{o} + \frac{1}{r_{n}I_{TG}}(T_{T} - T_{To}) \\ - \frac{1}{r_{n}^{2}I_{TG}}\frac{(T_{s} - T_{so})}{r_{F}}$$
(24)

Since the acceleration of transmission output shaft can be given as

$$\beta_M I_M \alpha_M = \frac{1}{r_n I_{TG}} T_T - \frac{1}{r_n^2 I_{TG}} \frac{T_s}{r_F} - \dot{\omega}_s,$$
  
$$\beta_M = \frac{1}{I_M} + \frac{1}{r_n^2 I_{TG}}$$

and, from Eqs. (4)-(5), it may be seen that smooth change of  $\omega_M$  implies small variation of the shaft torque and consequently a small vehicle jerk. Thus, the curve (24) implies a smooth change in  $\alpha_M$ . In other words, the shaft torque is considered to be a load on the transmission gear, instead of the whole power-train. These facts explain the validity of the curve (24).

Although the curve (23) or (24) implies a smooth acceleration change, it doesn't satisfy the dynamic shift condition. To satisty the condition, an equivalent acceleration term should be known. Generally, the term  $T_T - T_{To}$  is positive due to the torque amplifying effect of the torque converter. The shaft torque depends both on the load and clutch torque itself. These two terms can not be predicted in advance. As a result, shifting may not be accomplished within the designated shift time. One way to treat this problem is through an adoption of a *self-zero-approaching function* when the slip speed reaches a certain small critical value,  $\omega_{s,cr}$ . Examples of such function are constructed as follows

$$\dot{\omega}_{s} = \dot{\omega}_{so} \left( |\omega_{s}| / \omega_{s,cr} \right)^{m}$$

$$\dot{\omega}_{s} = \dot{\omega}_{so} \left( 1 + \cos\left( \pi \left( 1 - |\omega_{s} / \omega_{s,cr}| \right)^{m} \right) \right) / 2$$
(25)

Another example of a self-zero-approaching function is the curve (21) whose independent variable, time, is replaced with  $|1 - \omega_s/\omega_{s,cr}|^m$ . At the termination phase of shifting, these functions ensure that the desired slip speed approaches zero eventually.

The clutch torque for the indirect smooth acceleration shifting can be given from the relation (26) which was derived as in Eq. (9)

$$\dot{\omega}_{s} = \frac{1}{r_{n}I_{TG}}T_{T} + \frac{1}{I_{M}}\frac{T_{s}}{r_{F}} - \beta_{M}T_{Cn} \qquad (26)$$

#### 5.2 Simulation results

The curve (24) serves as a desired reference trajectory. Supposing that suitable information on the turbine torque and shaft torque is available, the relation (26) provides a control input torque for smooth acceleration shift. Based on Eqs. (24)-(26), an indirect smooth acceleration shift was simulated as shown in Fig. 6. Here, the effects of dynamic torque converter and torsional stiff-



Fig. 6 Indirect smooth acceleration shift

ness of the axle shaft ( $K_s$ =6742 Nm/rad) are included. The driving torque,  $T_{ENG}$ , and load torque,  $T_L$ , are assumed to be constant. Engine firing fluctuations were neglected.

As shown in the figure, acceleration of the transmission output and vehicle vary smoothly. It means that the jerk is small as expected. Although the shift completion time was set at 2 sec. for good visibility, any realistic shift time should not alter the fundamental results. Note that the actual shifting was completed a little earlier than the designated time due to the dynamic turbine torque. After 6.5 sec. the response is mainly governed by the self-zero-approaching curve. In order to improve the responses, one may try other curves or tuning the curve parameters. However, there is a limitation since oscillation is induced from the torsional spring effect of the axle shaft. In this view, the axle shaft should be as stiff as possible.

#### 6. Conclusions

In order to provide a basic strategy for a smooth shift control system of an automatic power transmission by using RPM sensors only and to accomplish the shift within designated time, detailed characteristics of the smooth shifting for clutch-to-clutch shift mechanism were studied in this paper. Several relationships involving key variables of the slip speed, vehicle acceleration, engaged clutch torque and heat dissipation were derived under simplifying assumptions. A dynamic shift condition to be met for accomplishing the shift process was established. In order to reduce the heat dissipation, it was shown that decreasing the pressure with time is more desirable than increasing it with time. Unfortunately, the latter is generally used in production vehicles.

Three types of smooth shifting, namely the slip shift, clutch torque shift and acceleration shift were considered. And smooth curves for these shifts were analyzed to accomplish the shift process within a designated shift completion time. Finally, the desired slip speed trajectory was proposed for indirect smooth acceleration shift, which incorporates a self-zero-approaching curve to fulfill the dynamic shift condition. The clutch torque for this shift was also provided under the assumption that the turbine and shaft torque information is available. Designing estimators for obtaining the necessary information will be presented in the near future (Jeong and Lee, 1999).

# Acknowledgement

This work was supported by the Korea Research Foundation and partially supported by KOSEF PDoc fellowship program.

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